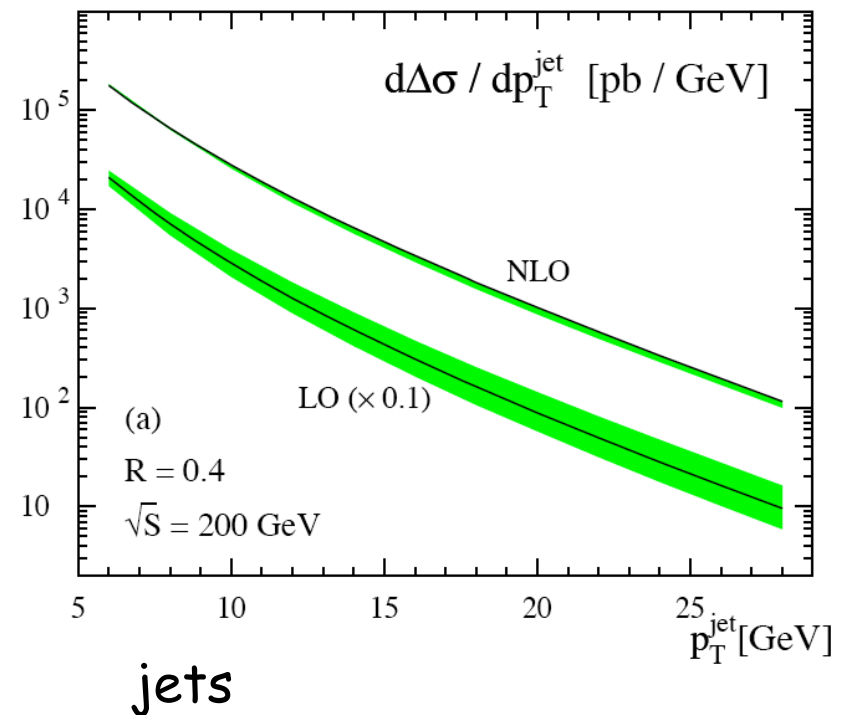
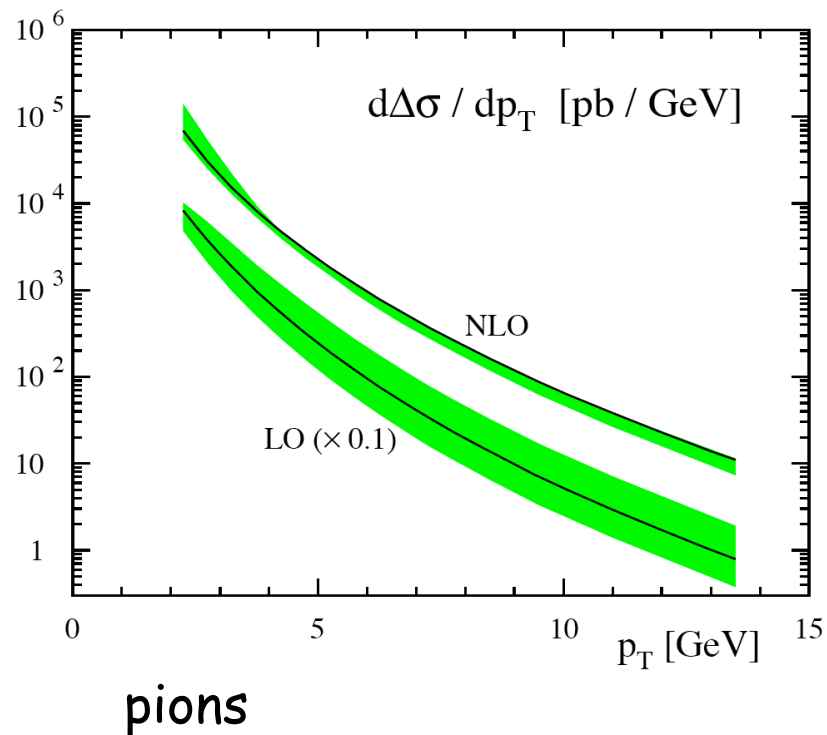


# Some remarks about theory uncertainties in extractions of polarized pdfs

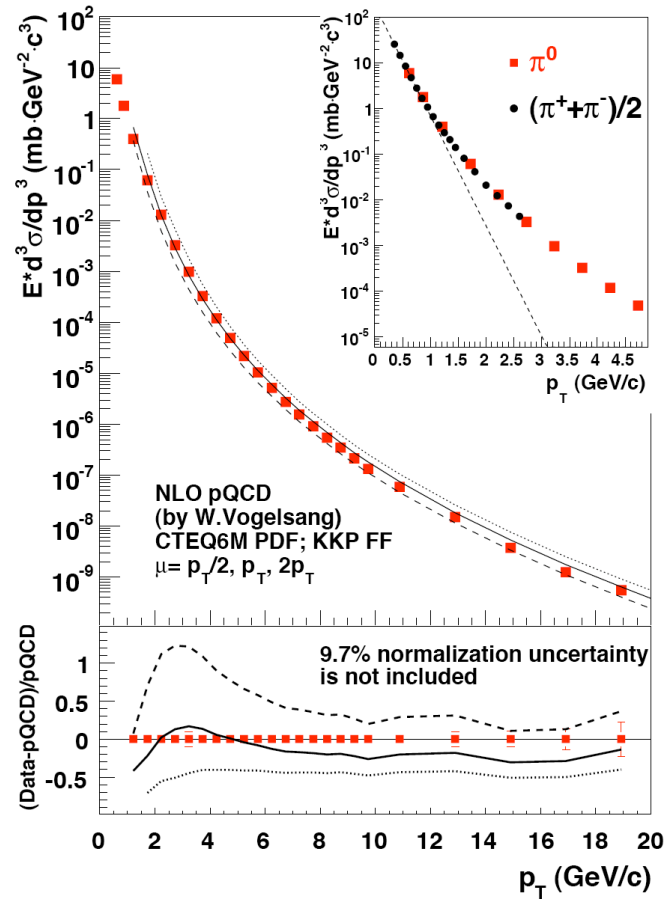
Werner Vogelsang  
Nuclear Theory, BNL

## Scale dependence:

- typically significant in hadronic collisions
- (some) decrease from LO to NLO
- often better in in polarized case:



- Use unpolarized cross section for guidance on scale choice

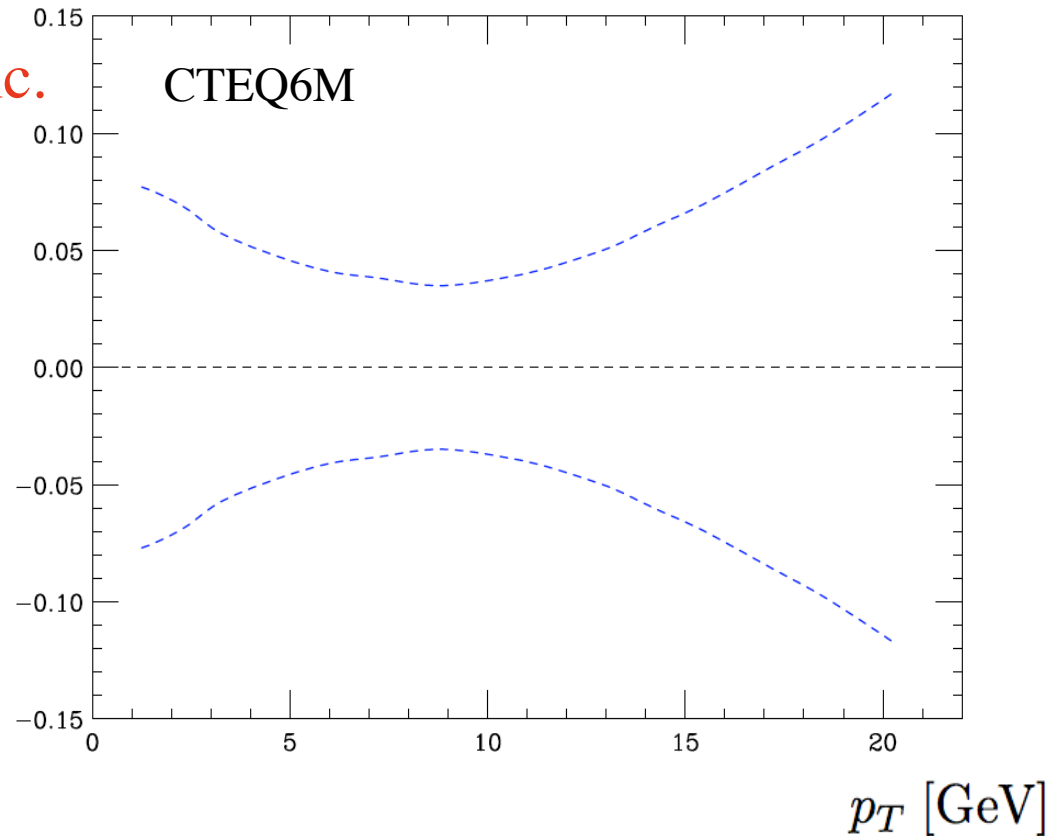


- Provide polarized cross section from experiment
- Resummation of higher orders

Bunce

- Unpolarized pdfs:  $pp \rightarrow \gamma X$  RHIC

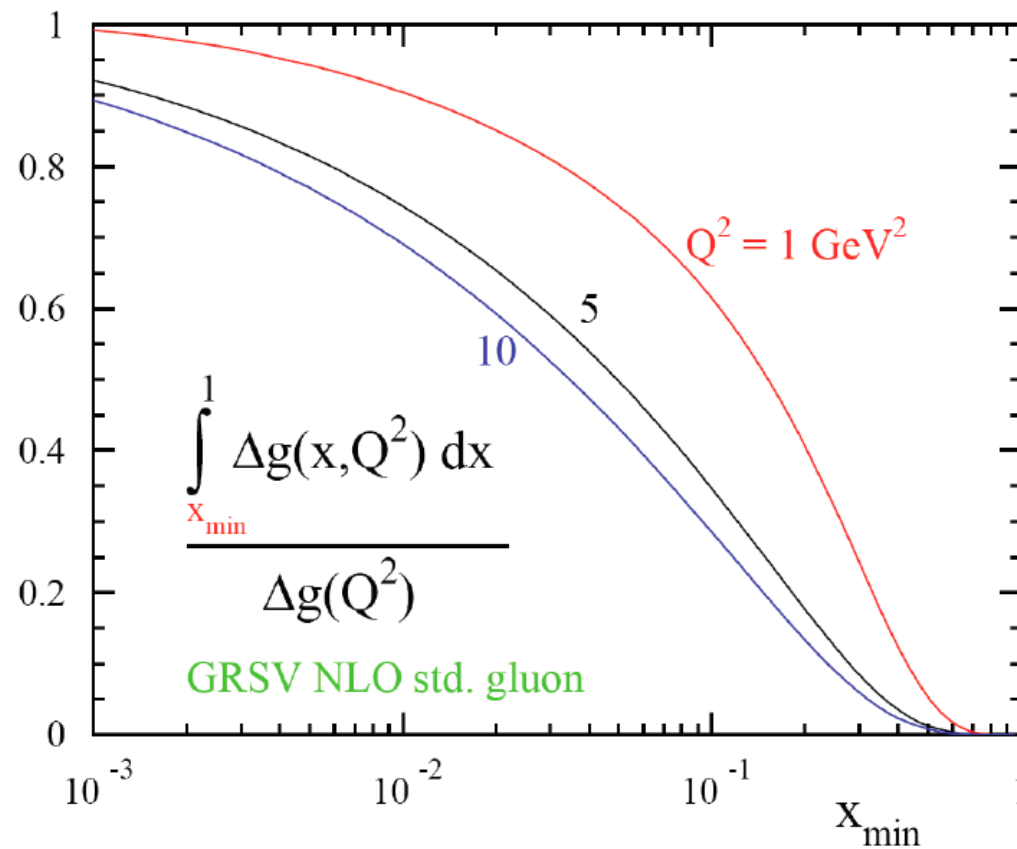
Relative unc.



- Fragmentation functions
- Strong coupling  $\alpha_S$
- Ideally, use all in one global fit ...

## Small-x behavior of pdfs :

- Interesting by its own right
- May be crucial for first moments



## Approaches:

- Perturbative: evolution, resummation
- Non-perturbative: models

- what does **DGLAP** evolution tell us ?

$$\mu \frac{d}{d\mu} \begin{pmatrix} \Delta \Sigma(x, \mu^2) \\ \Delta g(x, \mu^2) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta \mathcal{P}_{qq} & \Delta \mathcal{P}_{qg} \\ \Delta \mathcal{P}_{gq} & \Delta \mathcal{P}_{gg} \end{pmatrix}_{(z, \alpha_s(\mu))} \cdot \begin{pmatrix} \Delta \Sigma \\ \Delta g \end{pmatrix} \left( \frac{x}{z}, \mu^2 \right)$$

- at  $x \rightarrow 0$  (and to lowest order):

$$\begin{pmatrix} \Delta \mathcal{P}_{qq} & \Delta \mathcal{P}_{qg} \\ \Delta \mathcal{P}_{gq} & \Delta \mathcal{P}_{gg} \end{pmatrix} (x, \alpha_s) \approx \frac{\alpha_s}{2\pi} \begin{pmatrix} C_F & -n_f \\ 2C_F & 4C_A \end{pmatrix} + \mathcal{O}(x)$$

- compare unpolarized case:

$$\begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix} \approx \frac{\alpha_s}{2\pi} \frac{1}{x} \begin{pmatrix} 0 & 0 \\ 2C_F & 2C_A \end{pmatrix} + \mathcal{O}(1)$$

- solution (double-log approx. DLA)

Berera  
Ball, Forte, Ridolfi  
Gehrmann, Stirling

- interplay between **input** and **pert. evolution**:

- \* “flat” input : small-x behavior driven by evolution

$$\Delta g(x, Q^2) \propto \frac{1}{\sqrt{4\pi\gamma_+ \sqrt{\ln \frac{x_0}{x} \ln \frac{t}{t_0}}}} \exp \left[ 2\gamma_+ \sqrt{\ln \frac{x_0}{x} \ln \frac{t}{t_0}} - \delta \ln \frac{t}{t_0} \right]$$

$\gamma_+ \approx 2.5$

- \* power-like input  $\Delta\Sigma, \Delta g \sim x^{-\alpha}$   
preserved under evolution -- rise indep. of  $Q^2$

- for first case:

$$\frac{\Delta g(x, Q^2)}{xg(x, Q^2)} \propto \exp \left[ 2(\gamma_+ - \gamma_u) \sqrt{\ln \frac{x_0}{x} \ln \frac{t}{t_0}} + \dots \right]$$

- higher orders in the splitting functions at small  $x$ :

$$\Delta P(x) = \frac{\alpha_s}{2\pi} c_0 + \left(\frac{\alpha_s}{2\pi}\right)^2 c_1 \ln^2 x + \dots + \left(\frac{\alpha_s}{2\pi}\right)^{k+1} c_k \ln^{2k} x + \dots$$

- compare unpolarized case: (gq, gg spl. fcts.)

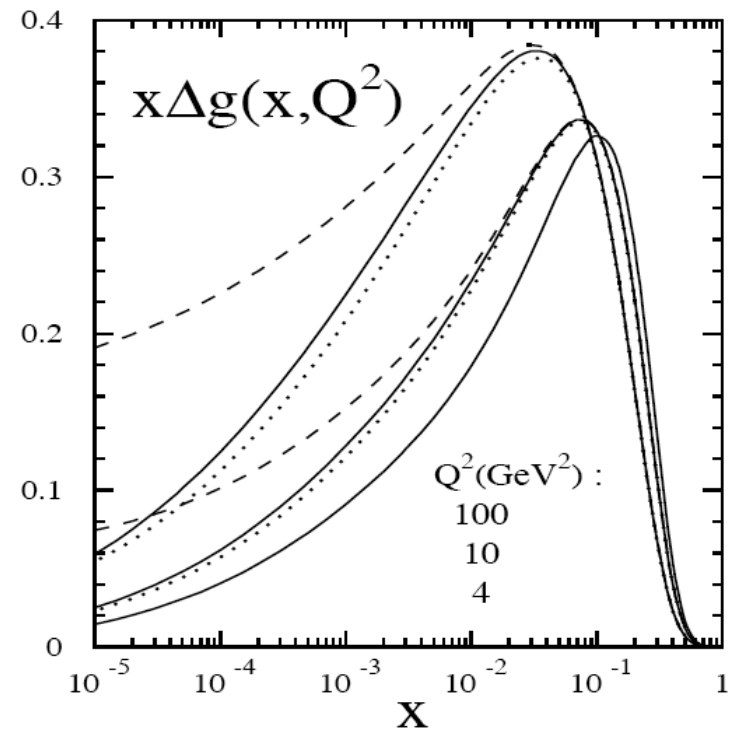
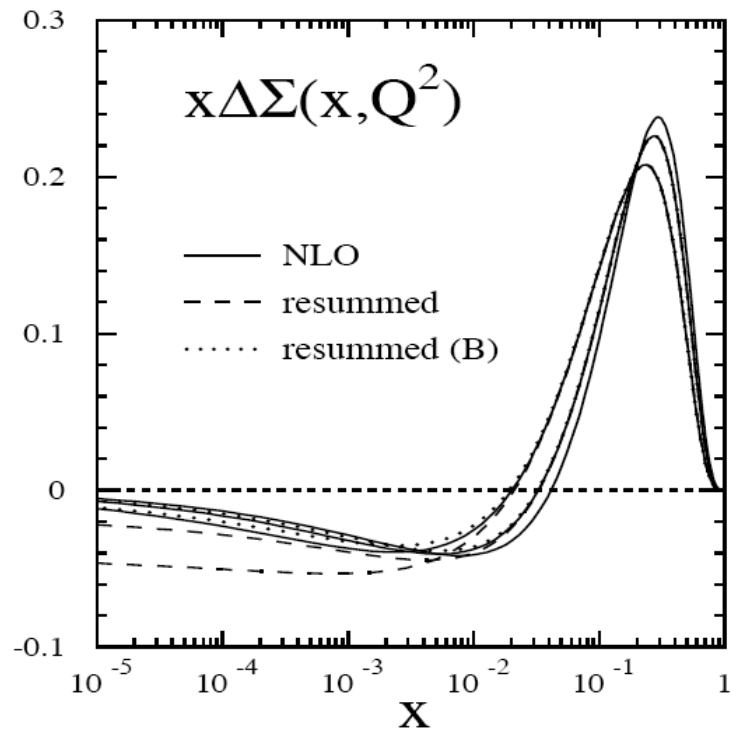
$$P(x) = \frac{\alpha_s}{2\pi} \frac{c'_0}{x} + \left(\frac{\alpha_s}{2\pi}\right)^2 c'_1 \frac{\ln x}{x} + \dots + \left(\frac{\alpha_s}{2\pi}\right)^{k+1} c'_k \frac{\ln^k x}{x} + \dots$$

- all-order resummation in polarized case:

Kirschner,Lipatov; Bartels,Ermolaev,Ryskin; Kwiecinski,Ziaja; Ermolaev,Greco,Troyan; Maul

- typically predict steep power-like rise
- however, subleading terms probably remain crucial

Blümlein,Riemersma,Vogt

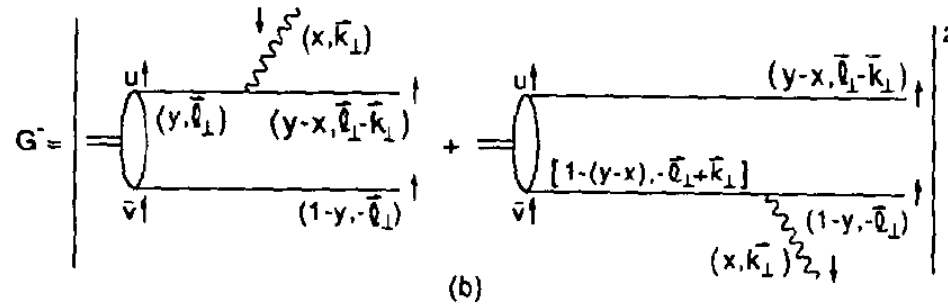
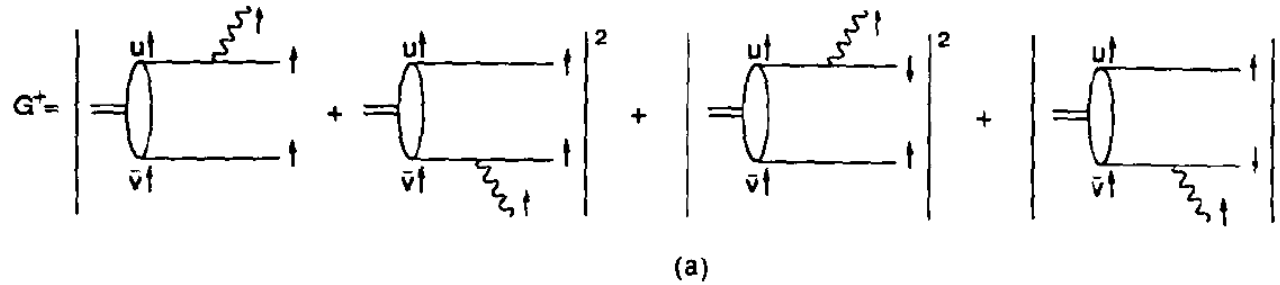


Blümlein, Riemersma, Vogt

- model approach:

Brodsky, Schmidt

Positronium



- argue that

$$\Delta g(x) \approx xg(x) \quad x \rightarrow 0$$

- CTEQ6M:

$$\int_0^{0.01} dx x g(x, Q^2 = 10) = 0.065$$

$$\int_{0.3}^1 dx g(x, Q^2 = 10) = 0.096$$

$$\int \Delta g \text{ [DSSV]}$$

$$-0.1$$

$$0.017$$