beam line project

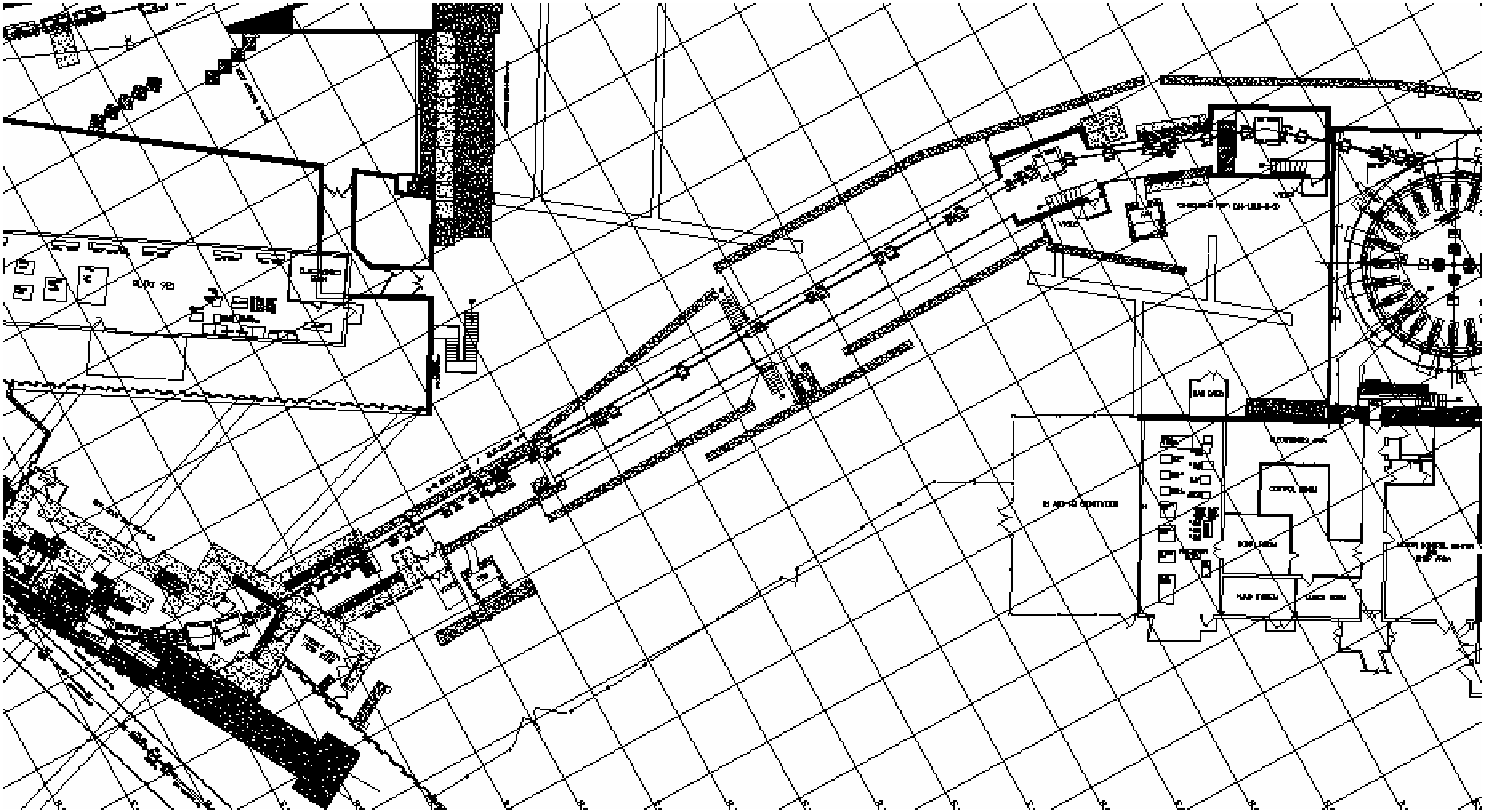
g-2 is statistics limited
g-2 needs more muons

goal x4 muons

items under consideration
- **target**
- **capture optics**
- **decay channel**
- **backward decays**
- **inflector**
- **…**

electronic notebook at http://zero.npl.uiuc.edu:8081

V line V target to g-2 ring



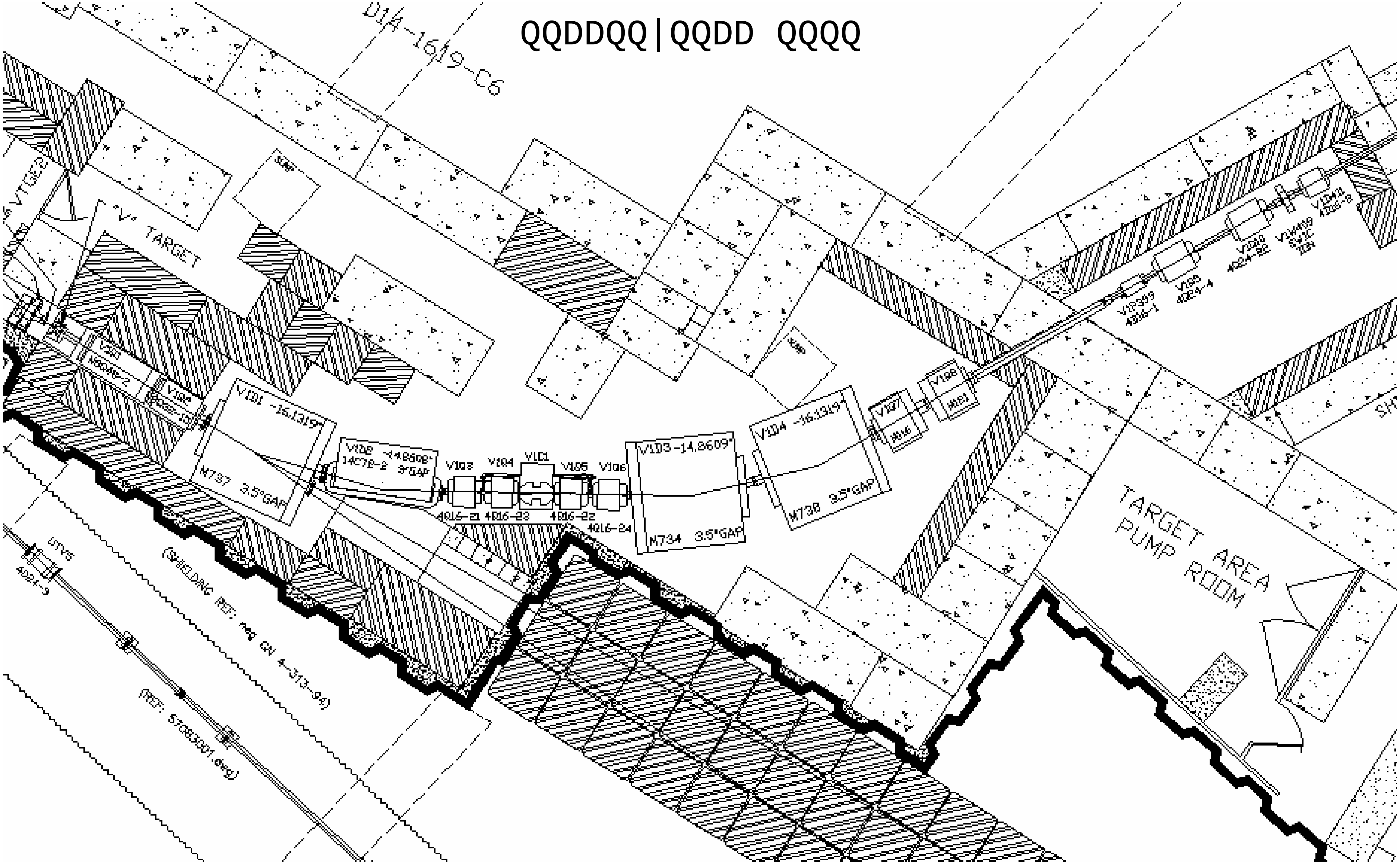
6 dipoles
29 quads

g2pimu.inp and Design Report

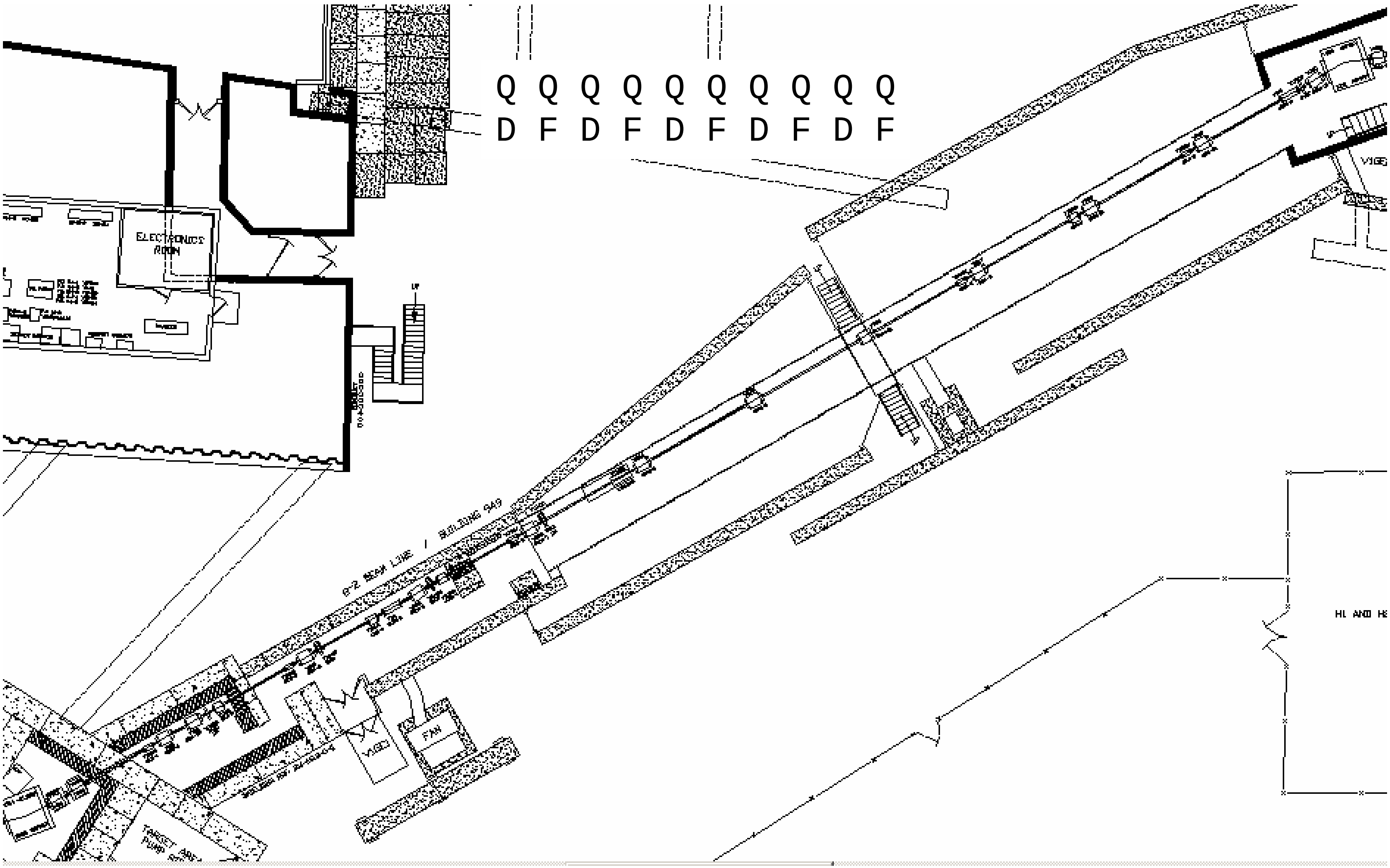
magnet	type	B(kG)	G(kG/in)	Quad radius (in)	field at pole (kG)	Leff (in)	I(kA)	R(mohm)	V (V)	P (kW)
V1Q1	8Q48		-3.349	3.750	-12.558	52.00	2.779	35.0	97	270
V1Q2	8Q32		2.657	3.750	9.965	36.00	2.118	25.6	54	115
V1D1	6X18D72	-15.039				75.00	0.850	46.0	72	114
V1D2	6X18C72	-14.154				75.00	1.300	35.0	46	61
V1Q3	4Q16		-3.500	1.875	-6.563	18.00	0.280	183.0	51	14
V1Q4	4Q16		3.555	1.875	6.667	18.00	0.307	183.0	56	17
V1Q5	4Q16		3.555	1.875	6.667	18.00	0.307	183.0	56	17
V1Q6	4Q16		-3.500	1.875	-6.563	18.00	0.230	183.0	51	14
V1D3	3X18D72	-14.182				75.00	0.921	46.0	33	24
V1D4	3X18D72	-15.387				75.00	0.990	46.0	36	30
V1Q7	8Q13		2.092	3.750	7.845	28.00	1.592	21.4	34	54
V1Q8	8Q13		-2.166	3.750	-8.123	28.00	1.676	21.4	35	60
(VS1)	4D16		0.610			16.00	0.300	500.0	15	0.5
V1Q9	4Q24		0.558	1.875	1.046	26.00	0.131	30.4	4	0.5
V1Q10	4Q24		-0.555	1.875	-1.041	26.00	0.157	30.4	4	0.7
V1Q11	4Q24		-0.954	1.875	-1.788	26.00	0.123	30.4	3	0.5
V1P1	5D22	-4.809				36.00	0.676	40.0	27	18
V1Q12	8Q24		0.954	1.875	1.788	26.00	0.123	30.4	3	0.5
V1Q13	4Q24		-0.954	1.875	-1.788	26.00	0.123	30.4	3	0.5
V1Q14	4Q24		0.954	1.875	1.788	26.00	0.123	30.4	3	0.5
V1Q15	4Q24		-0.954	1.875	-1.788	26.00	0.123	30.4	3	0.5
V1Q16	4Q24		0.954	1.875	1.788	26.00	0.123	30.4	3	0.5
V1Q17	4Q24		-0.954	1.875	-1.788	26.00	0.123	30.4	3	0.5
V1Q18	4Q24		0.954	1.875	1.788	26.00	0.123	30.4	3	0.5
V1Q19	4Q24		-0.954	1.875	-1.788	26.00	0.123	30.4	3	0.5
V1P2	5D22	-4.809				36.00	0.676	40.0	27	18
V1Q20	4Q24		0.477	1.875	0.894	26.00	0.061	30.4	1	0.1
V1D5	3X18D72	20.030				75.00	1.200	45.8	55	66
V1Q21	4Q24		0.859	1.875	1.611	26.00	0.110	30.4	3	0.54
V1Q22	4Q24		-1.222	1.875	-2.291	26.00	0.155	30.4	3	0.4
V1Q23	4Q24		1.429	1.875	2.679	26.00	0.181	30.4	5	1
V1Q24	4Q24		-1.222	1.875	-2.291	26.00	0.155	30.4	4	0.7
V1Q25	4Q24		0.859	1.875	1.611	26.00	0.110	30.4	3	0.4
V1D6	3X18D72	20.030				75.00	1.200	45.8	55	66
V1Q26	4Q24		0.907	1.875	1.700	26.00	0.090	30.4	2	0.2
V1Q27	4Q24		0.784	1.875	1.471	26.00	0.039	30.4	1	0.1
VS6	4d16		0.610			16.00	0.030	500.0	15	0.5
V1Q28	8Q24		-2.682	3.875	-10.391	28.00	2.032	30.4	3	0.5
V1Q29	8Q24		2.765	3.875	10.714	28.00	2.255	30.4	3	0.5

V line V target to Q10

QQDDQQ|QQDD QQQQ

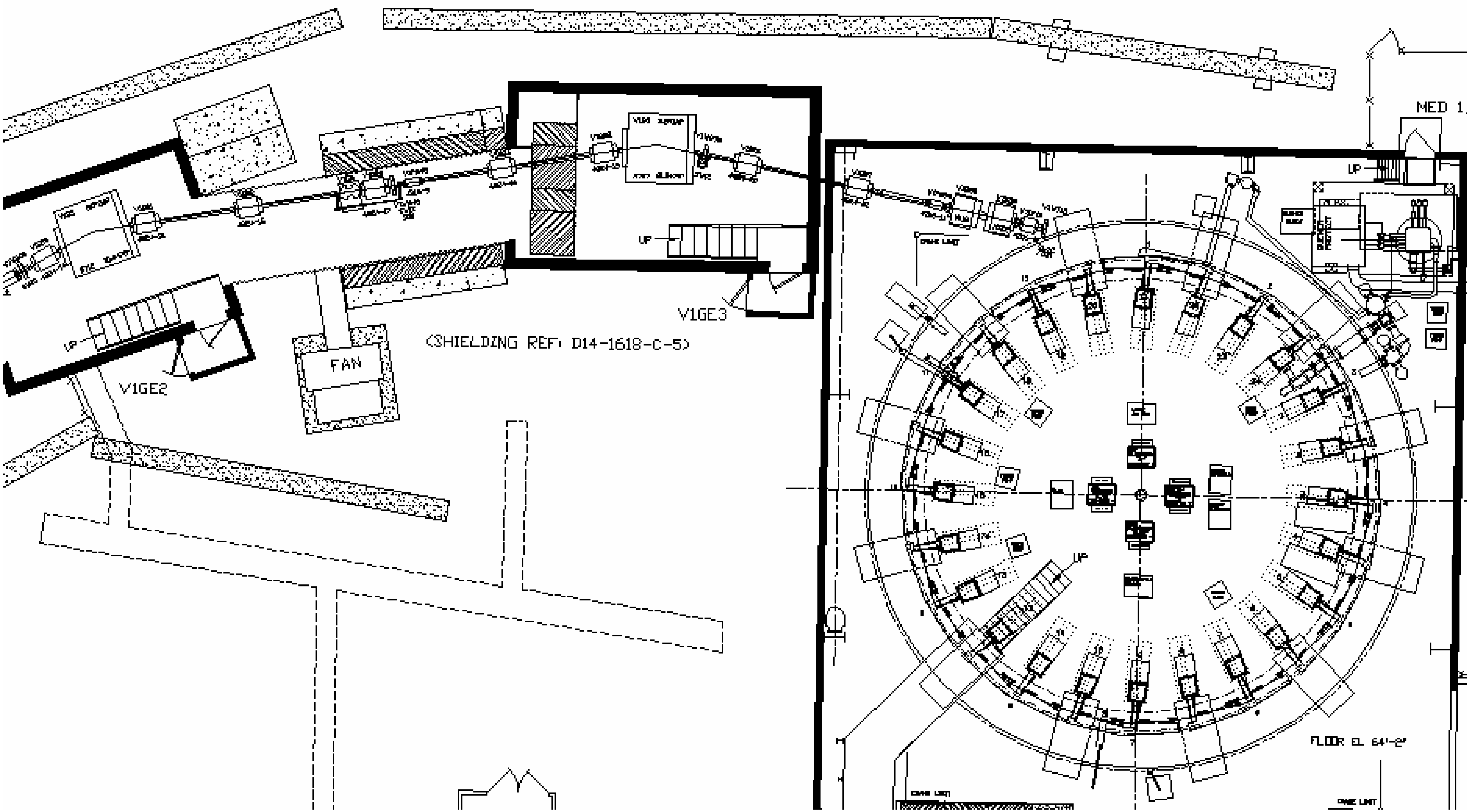


V line Q11 to Q 20



V line D5 to g-2 ring

DQQ Q QQD QQQQ



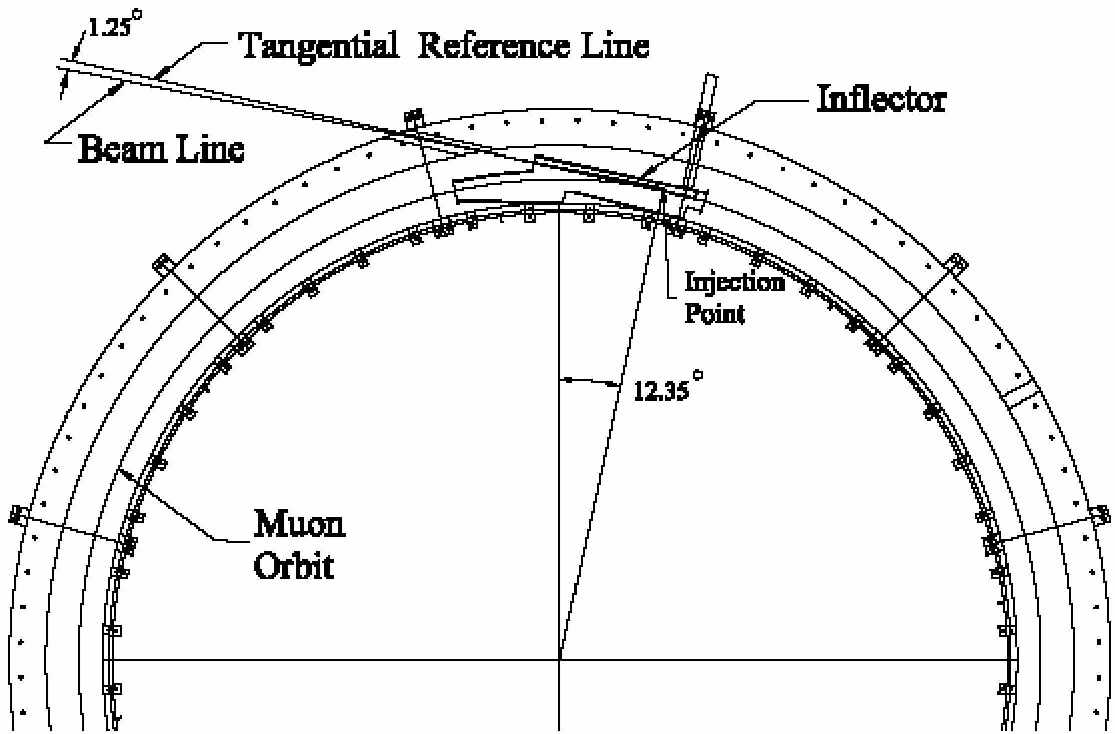


Fig. 1. Inflector in the storage ring.

inflector and storage ring apertures

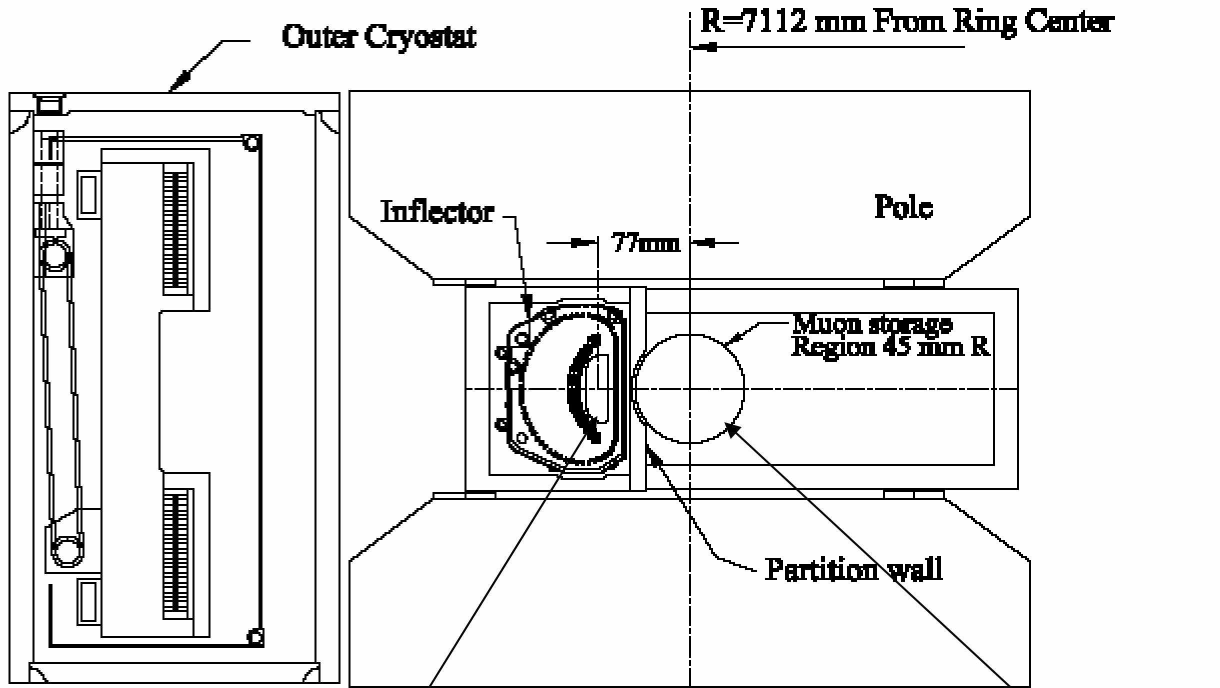


Fig. 4. Inflector at injection point.

TRANSPORT formalism I

first order TRANSPORT linearizes equations of motion

every beam line element is represented by a matrix

assuming a median plane transverse motions are uncoupled

useful to follow rays with $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or with $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$(X) = (R)(X_0) \qquad\qquad (Y) = (R)(Y_0)$$

$$\begin{pmatrix} x \\ \theta \\ \delta \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{16} \\ R_{21} & R_{22} & R_{26} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ \theta_0 \\ \delta \end{pmatrix} \qquad \begin{pmatrix} y \\ \varphi \end{pmatrix} = \begin{pmatrix} R_{33} & R_{34} \\ R_{43} & R_{44} \end{pmatrix} \begin{pmatrix} y_0 \\ \varphi_0 \end{pmatrix}$$

$$x = R_{11} x_0 + R_{11} \theta_0 + R_{16} \delta \qquad y = R_{33} y_0 + R_{34} \varphi_0$$

$$\theta = R_{21} x_0 + R_{21} \theta_0 + R_{26} \delta \qquad \varphi = R_{43} y_0 + R_{44} \varphi_0$$

$$\delta = \Delta p / p$$

TRANSPORT formalism II

beam is represented by ellipse in phase space

TRANSPORT of ellipse via same R matrix

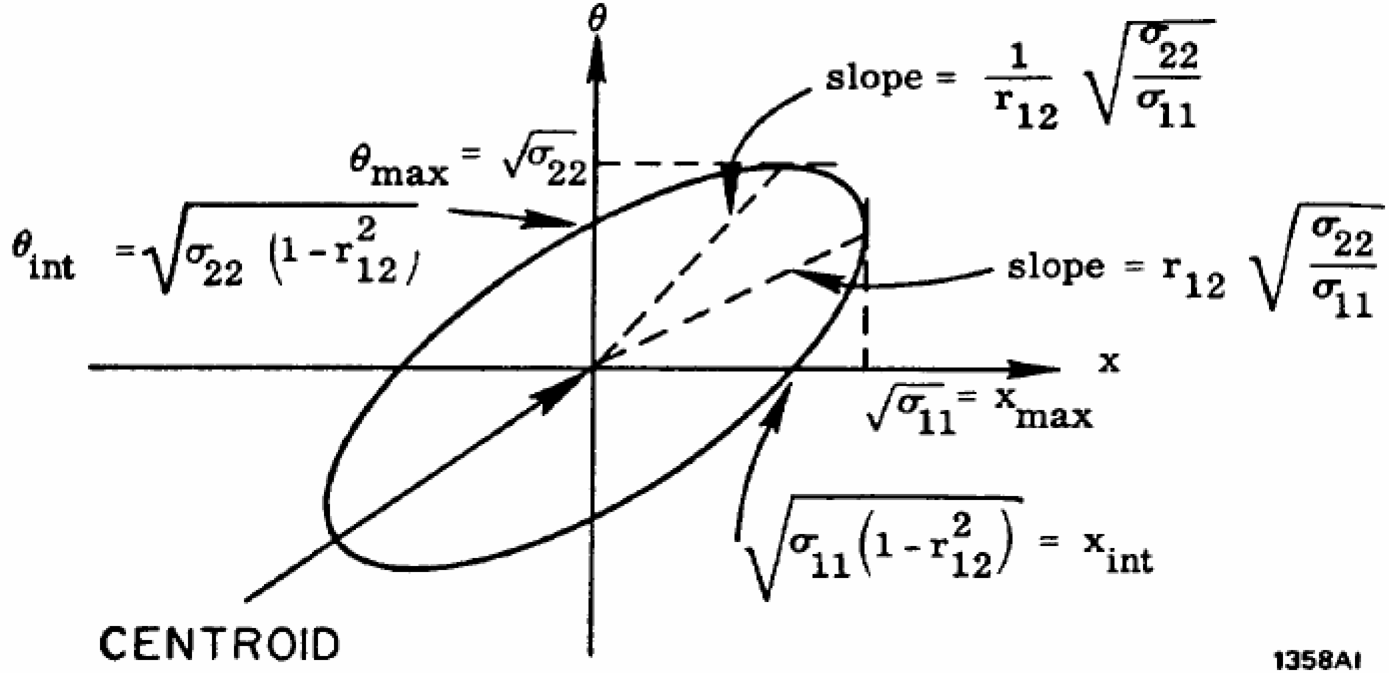
useful to follow ellipse or beam envelope

$$(\sigma) = \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \qquad (\sigma)^{-1} = \frac{1}{\varepsilon^2} \begin{pmatrix} \sigma_{22} & -\sigma_{21} \\ -\sigma_{21} & \sigma_{11} \end{pmatrix}$$

$$(x,\theta)(\sigma)^{-1}\begin{pmatrix} x \\ \theta \end{pmatrix} = 1 \qquad \sigma_{22}x^2 - 2\sigma_{21}x\theta + \sigma_{11}\theta^2 = \det(\sigma)$$

$$(\sigma_{new}) = (R)(\sigma_{initial})(R)^T$$

$$x_{max} = \sqrt{\sigma_{11}} \qquad \theta_{max} = \sqrt{\sigma_{22}}$$


θ
slope = $\frac{1}{r_{12}}\sqrt{\frac{\sigma_{22}}{\sigma_{11}}}$
$\theta_{max} = \sqrt{\sigma_{22}}$
$\theta_{int} = \sqrt{\sigma_{22}\left(1 - r_{12}^2\right)}$
slope = $r_{12}\sqrt{\frac{\sigma_{22}}{\sigma_{11}}}$
x
$\sqrt{\sigma_{11}} = x_{max}$
$\sqrt{\sigma_{11}\left(1 - r_{12}^2\right)} = x_{int}$
CENTROID
1358A1

TRANSPORT formalism III

beam ellipse can be expressed in terms of CSL parameters
often called accelerator notation

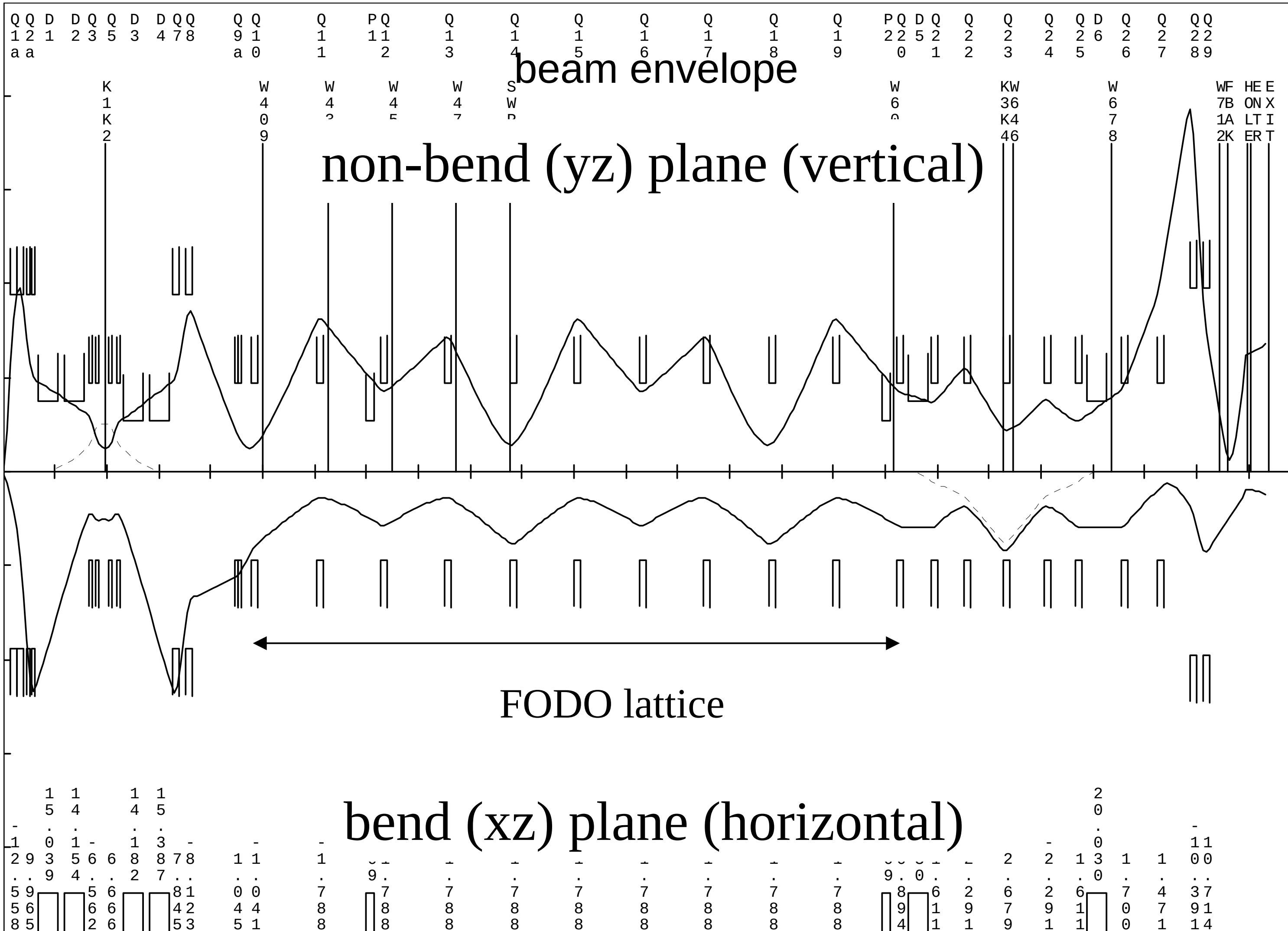
$$(\sigma) = \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\sigma_{11} = \varepsilon\,\beta$$

important relations: $\sigma_{11,\mathrm{max}} = \varepsilon\,\beta_{\mathrm{max}}$

$$x_{\mathrm{max}} = \sqrt{\varepsilon\,\beta_{\mathrm{max}}}$$

Transport calculation V target to g-2 ring
parameters from btraf g2pimu.inp



accelerator physics notation I
for FODO lattice

$$(X) = (R)^n (X_0)$$

$$(R)(X) = \lambda (X)$$

$$\det(R - \lambda I) = 0$$

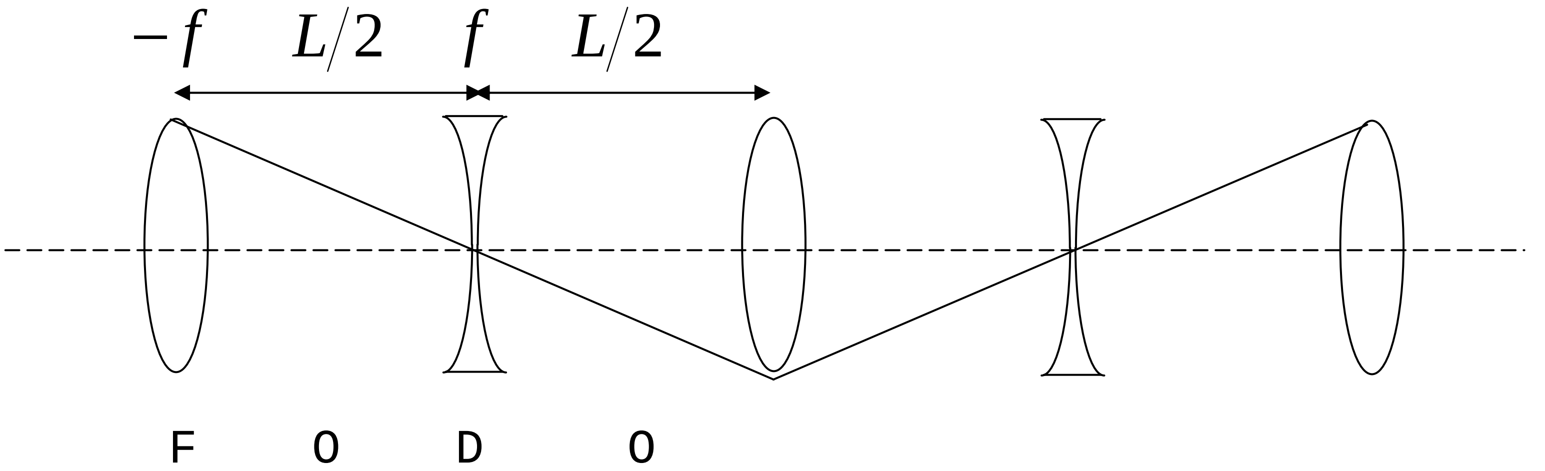
$$\lambda^2 - \lambda(R_{11} + R_{22}) + 1 = 0$$

$$\lambda = \left[(R_{11} + R_{22}) \pm \sqrt{(R_{11} + R_{22})^2 + 1} \right] \Big/ 2$$

λ real, one eigenvalue is > 1

for stability, λ must be complex

accelerator physics notation I
FODO lattice Transport matrix



$$\begin{pmatrix} 1 & \frac{L}{2} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}\begin{pmatrix} 1 & \frac{L}{2} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1+\frac{L}{2f} & L+\frac{L^2}{4f} \\ -\frac{L}{2f^2} & 1-\frac{L}{2f}-\frac{L^2}{4f^2} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

accelerator physics notation II
for FODO lattice

$$\lambda = \cos\mu \pm i \sin\mu$$

$$R = \begin{pmatrix} \cos\mu + \alpha \sin\mu & \beta \sin\mu \\ -\gamma \sin\mu & \cos\mu - \alpha \sin\mu \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\frac{1}{2}\mathrm{Tr}(R) = \cos\mu = 1 - \frac{L^2}{8f^2}$$

$$\sin\frac{\mu}{2} = \frac{L}{4f}$$

μ: phase advance

$$\left|\frac{L}{4f}\right| < 1$$

accelerator physics notation III
for FODO lattice

CSL parameters
(i.e. values of α, β, γ at F)

$$\alpha = \frac{a-d}{2\sin\mu} = \sqrt{\frac{1+\frac{L}{4f}}{1-\frac{L}{4f}}}$$

$$\gamma = -\frac{c}{\sin\mu} = \frac{1}{f}\sqrt{\frac{1}{1-\left(\frac{L}{4f}\right)^2}}$$

$$\beta = \frac{b}{\sin\mu} = 2f\sqrt{\frac{1+\frac{L}{4f}}{1-\frac{L}{4f}}}$$

$$\sin\frac{\mu}{2} = \frac{L}{4f}$$

accelerator physics notation IV
for FODO lattice
beta function

$$\beta_{\text{max}} = 2f\sqrt{\frac{1+\sin\frac{\mu}{2}}{1-\sin\frac{\mu}{2}}} \qquad \beta_{\text{min}} = 2f\sqrt{\frac{1-\sin\frac{\mu}{2}}{1+\sin\frac{\mu}{2}}}$$

$$\frac{1}{f} = \frac{g\ell}{(B\rho)} \qquad \sin\frac{\mu}{2} = \frac{L}{4f}$$

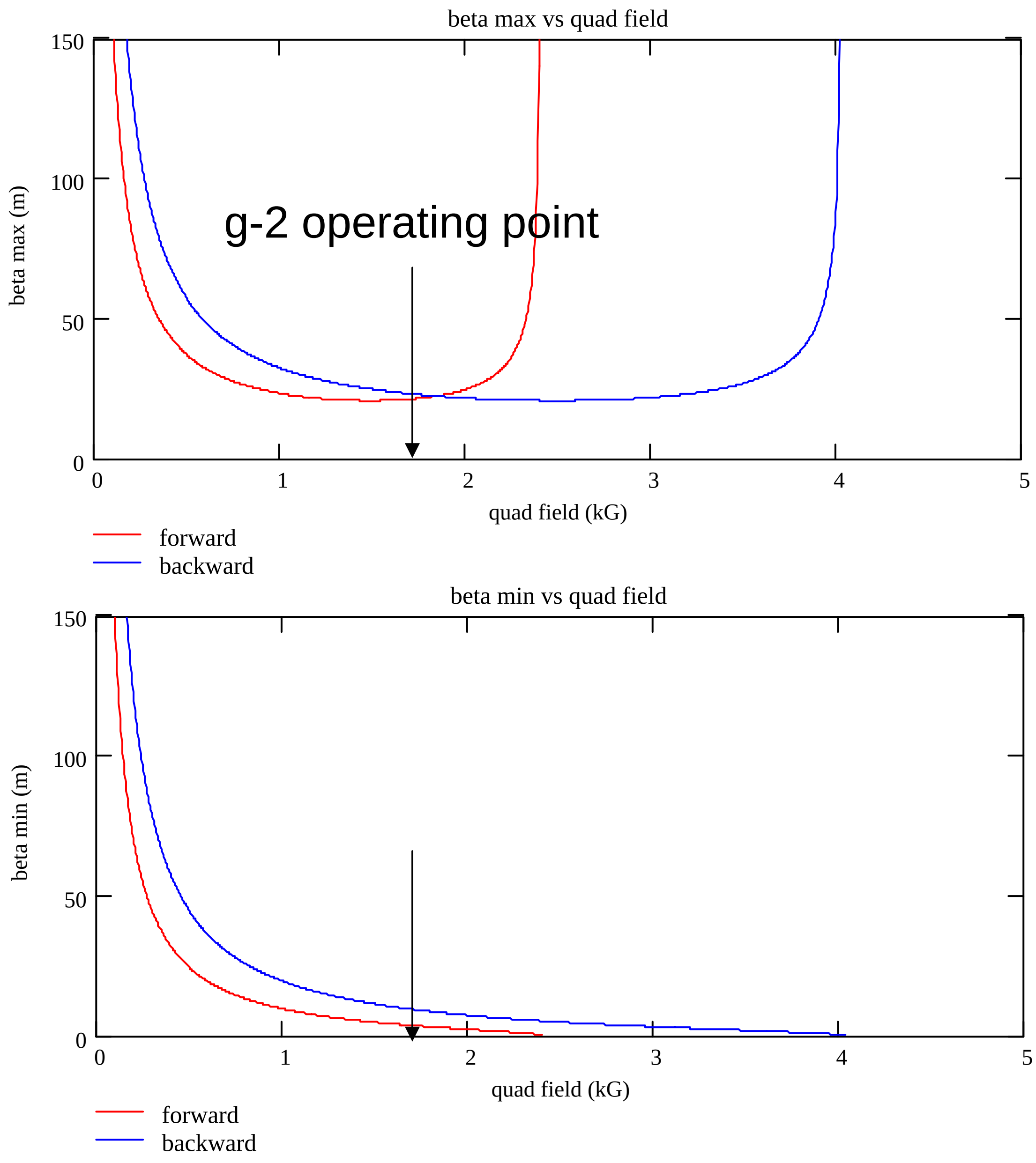
g	gradient
ℓ	length
$(B\rho)$	rigidity

max and min of beta function vs quad field

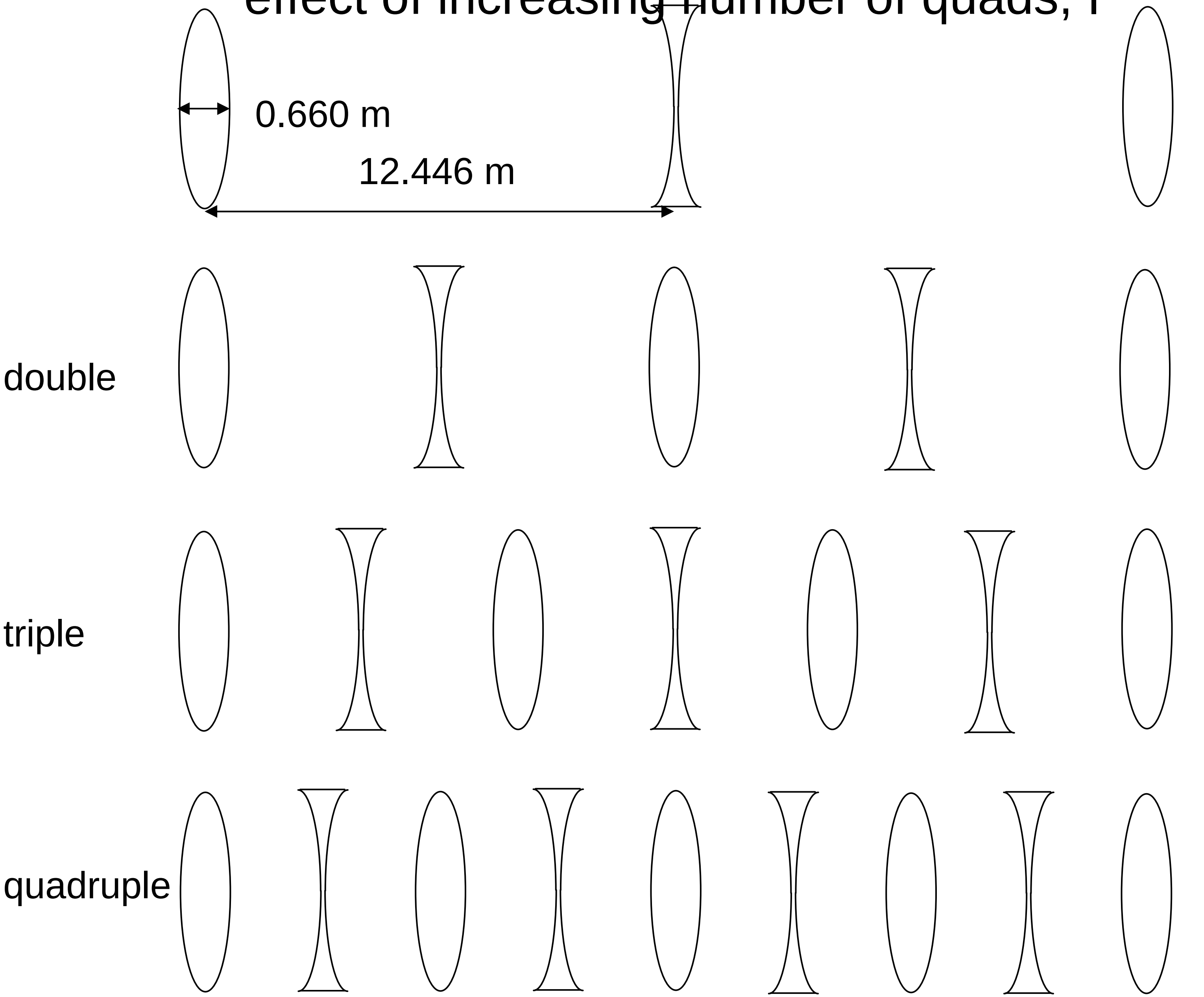
L = 12.446 m

forward 3.15 GeV/c

backward 5.22 GeV/c



effect of increasing number of quads, I



effect of increasing number of quads, II

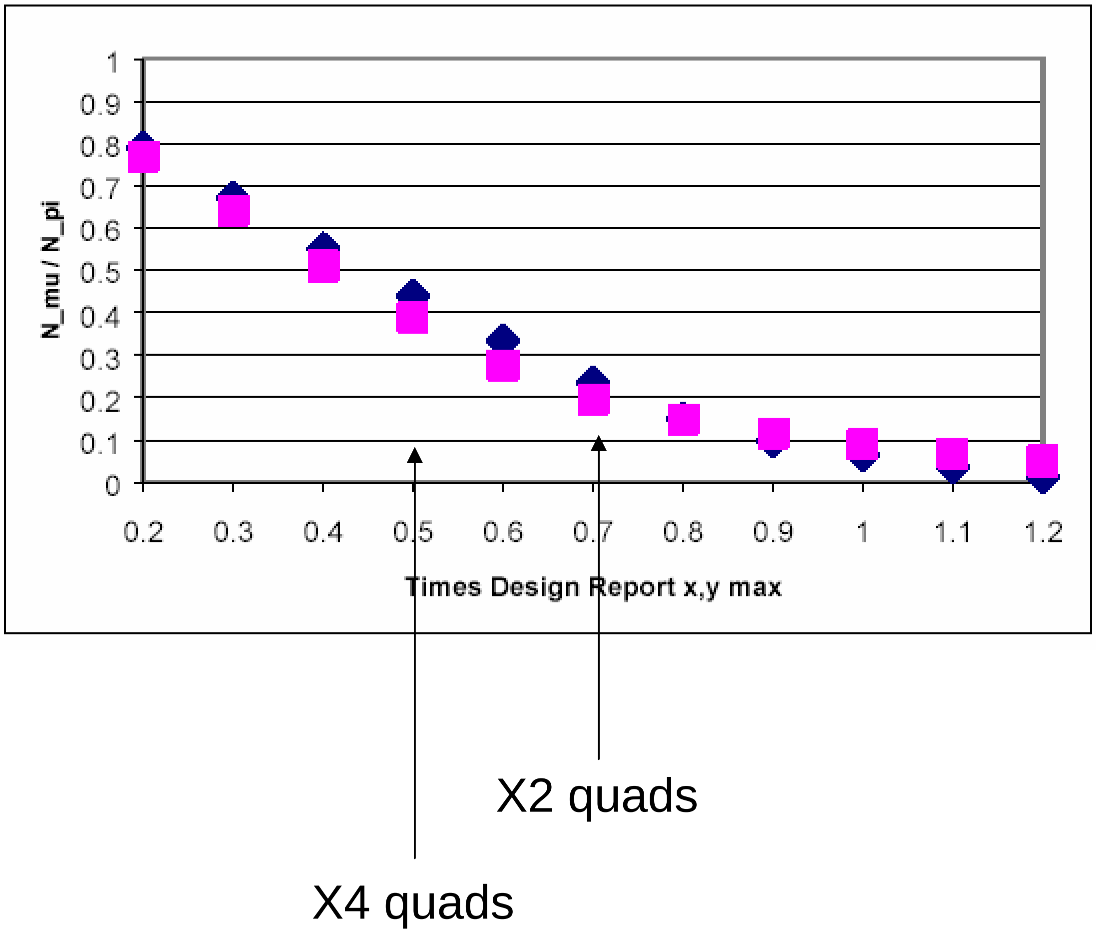
$$\sqrt{x^2{}_{\max,0}} = \sqrt{\varepsilon\,\beta_{\max,0}} \qquad x'_{\max,0} = x_{\max,0} \big/ \beta_{\max,0}$$

Suppose $\beta_{\max,0} \to \beta_{\max,0} \big/ 2$

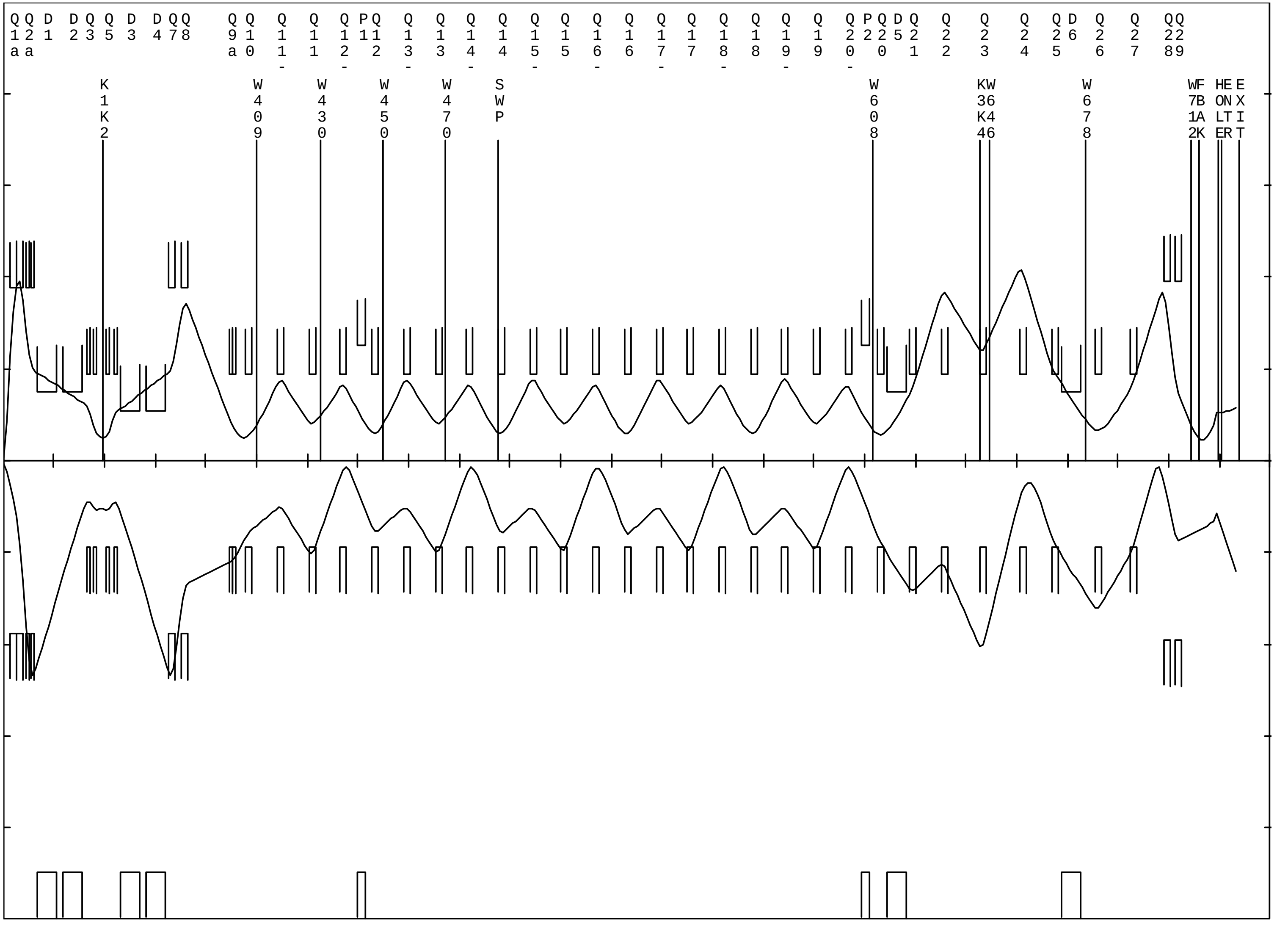
then $x_{\max,0} \to x_{\max,0} \big/ \sqrt{2}$ beam smaller

and $x'_{\max,0} = \sqrt{2}\; x'_{\max,0}$ divergence larger

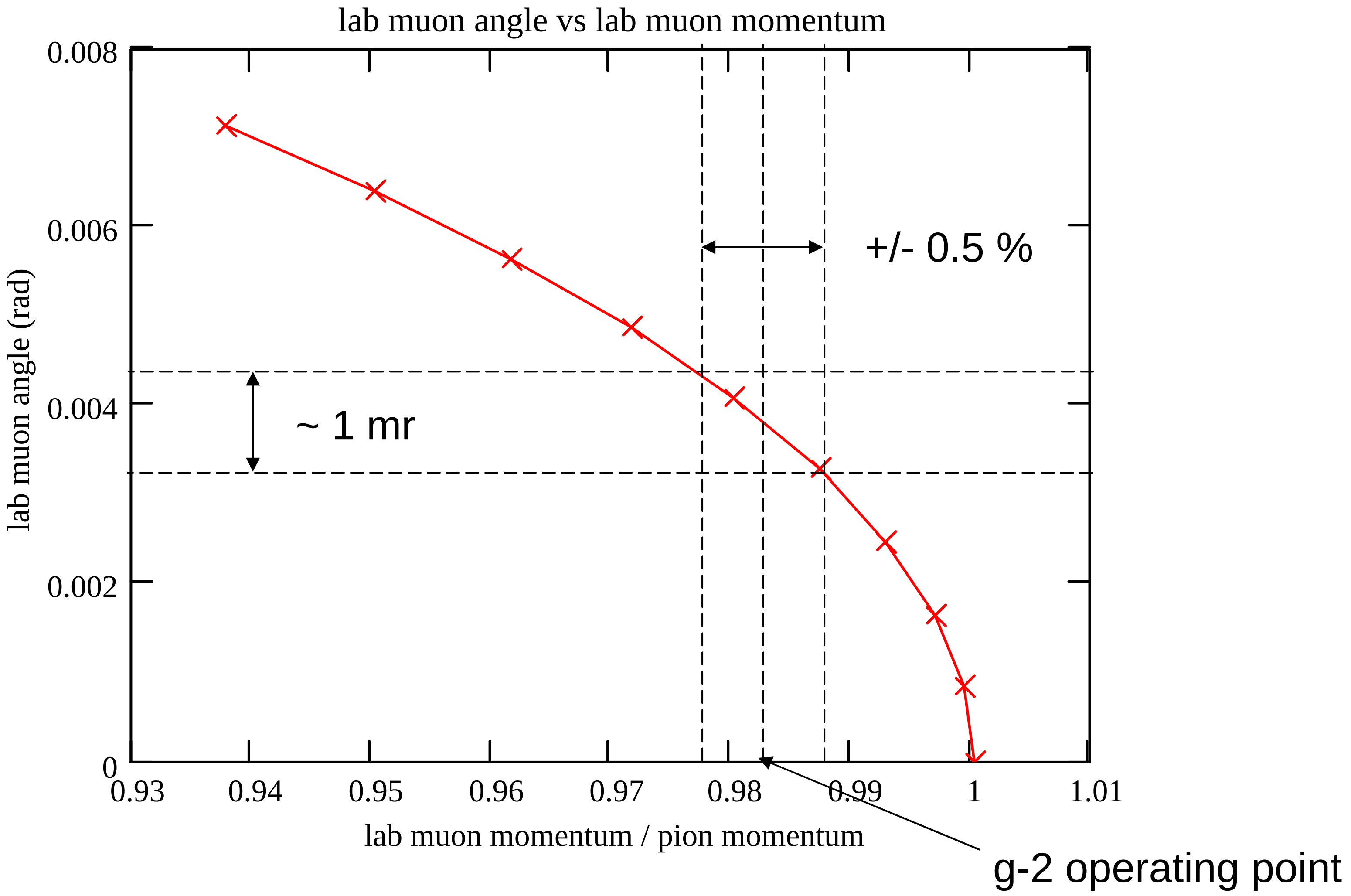
phase space calculation of effect of change in beta function
Morse g-2 #448



Transport calculation V target to g-2 ring
btraf g2pi.inp with doubled lattice



muon lab angle vs muon lab momentum



x at every five degrees in com

pion momentum, stored muons operating point

p_π / p_μ	e^+/SEC	F_π	A
1.005	179	80 %	0.22
1.010	77	30 %	0.26
1.015	37	6.5 %	0.30
1.017	30	1.6 %	0.30
1.020	22	0.9 %	0.30

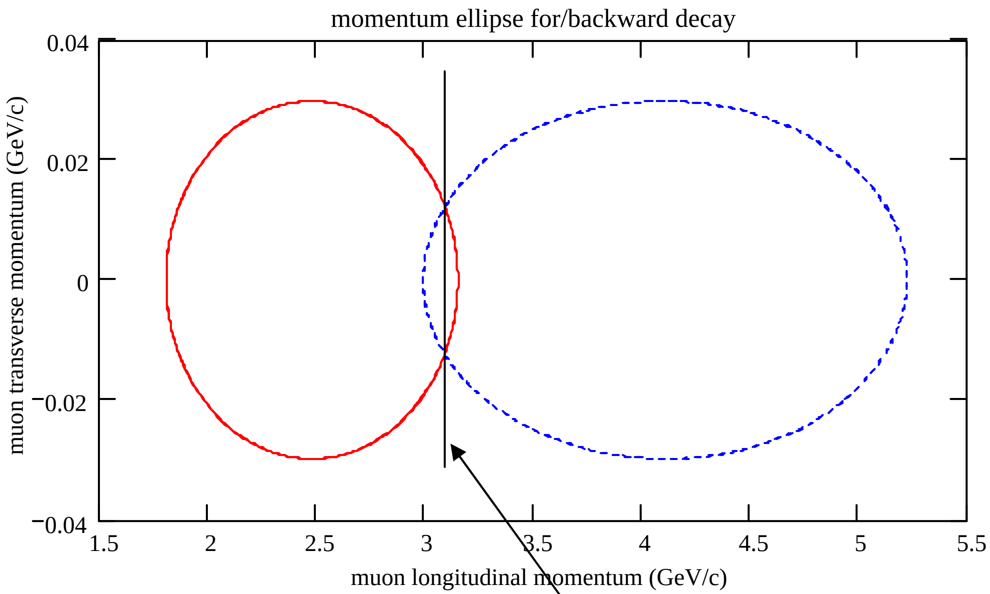
g-2 operating point → (row 1.017)

4 mr

source PRD draft

momentum ellipses for for/backward decays

p_{for} = 3.15 GeV/c p_{for} = 5.22 GeV/c



p_{magic}

what changes for backward decays?

simple scaling 5.22/3.11

	new B (kG) (dipole)	new g (kG/in) (quad)	new field at pole (kG) (quad)
V1Q1		-5.546	-20.797
V1Q2		4.403	16.510
V1D1	-24.901		
V1D2	-25.115		
V1Q3		-5.795	-10.866
V1Q4		5.887	11.039
V1Q5		5.887	11.039
V1Q6		-5.795	-10.866
V1D3	-23.483		
V1D4	-25.478		
V1Q7		3.459	12.971
V1Q8		-3.583	-13.436

possible factors improvement

increase number of quads in lattice x2

backward decays x4

open up inflector x1.7

goal x4 muons